Oracle design for Hamiltonian Cycle Problem using Grover's Algorithm

Diseño de oráculo del algoritmo de Grover para el problema del ciclo Hamiltoniano

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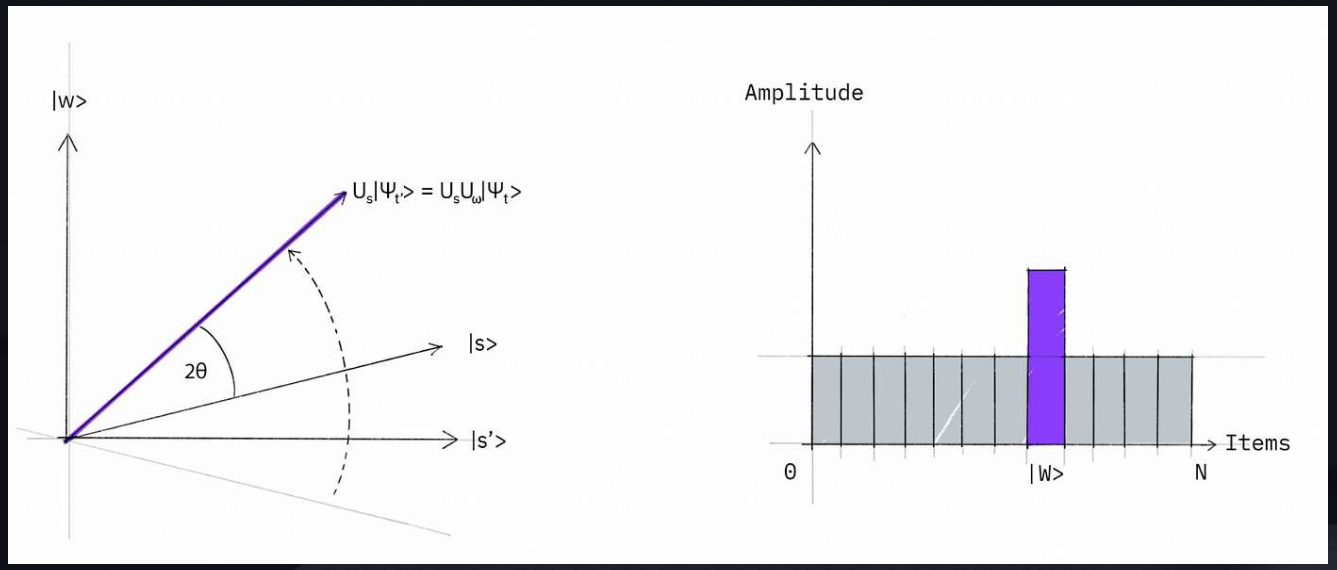
Content

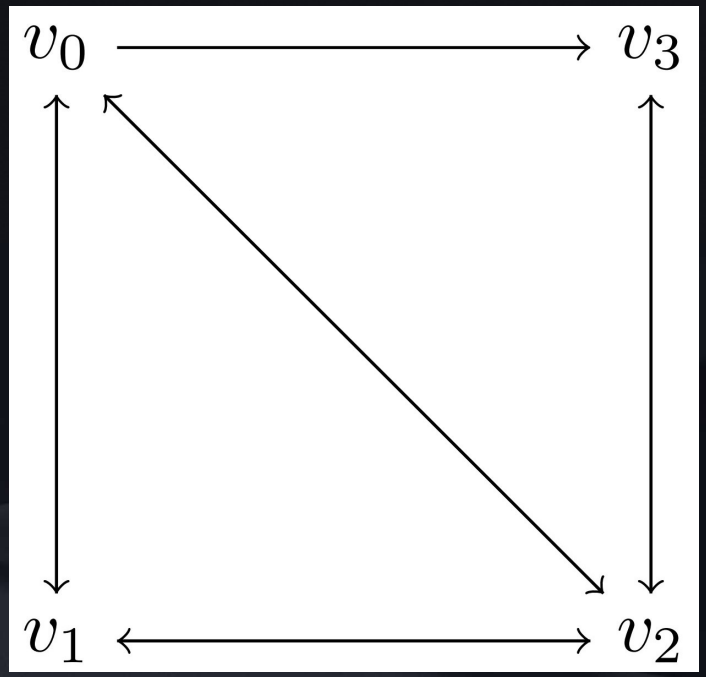
- Grover's algorithm overview
- Problem definition and encoding
- Oracle design for $N = 2^n$
- Generalized solution
- Conclusion

Grover's algorithm

- Search algorithm
- From $O(N)$ to $O(\sqrt{N})$
- Key parts:
  - Oracle
  - Diffuser



Problem definition and encoding



10-00-11-01

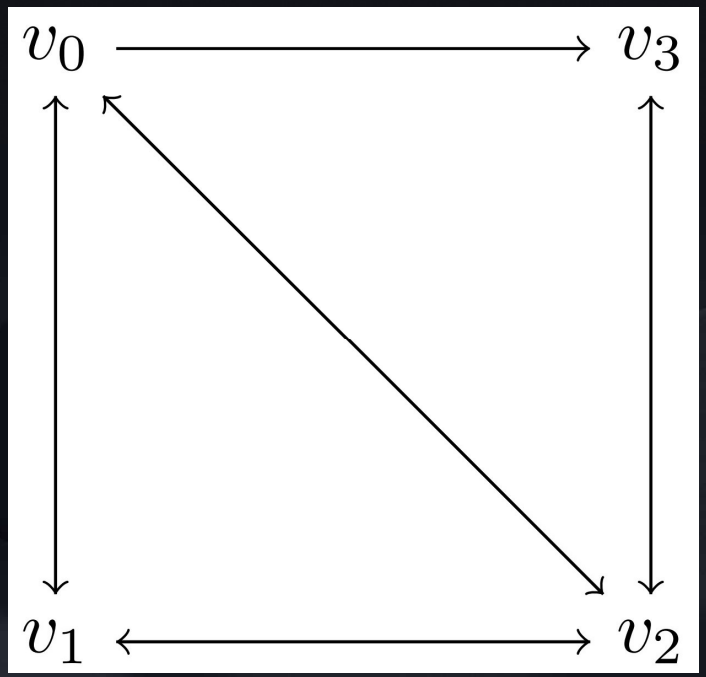
2 - 0 - 3 - 1

(V1-V3-V0-V2)

Constraints

- Each vertex should have a different position.
- Vertices with consecutive positions have to be connected by an edge. (The last and first positions are considered adjacent)

Bitstring	Integer representation	Validity	Cycle
00,01,10,01	0-1-2-1	Violates constraint 1	None
01,11,10,00	1-3-2-0	Violates constraint 2	(v_3, v_0, v_2, v_1)
00,11,10,01	0-3-2-1	Yes	(v_0, v_3, v_2, v_1)

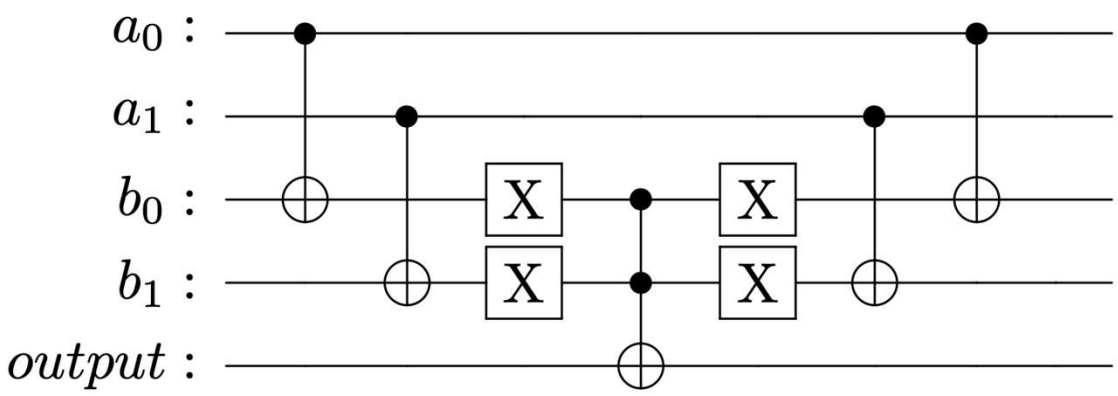


Circuit initialization

- Position index register (main register)
  - $N\log(N)$ qubits
- Unique position register (first constrain ancilla)
  - C_2^N qubits
- Adjacent vertex register (second constraint ancilla)
  - $O(N^2)$

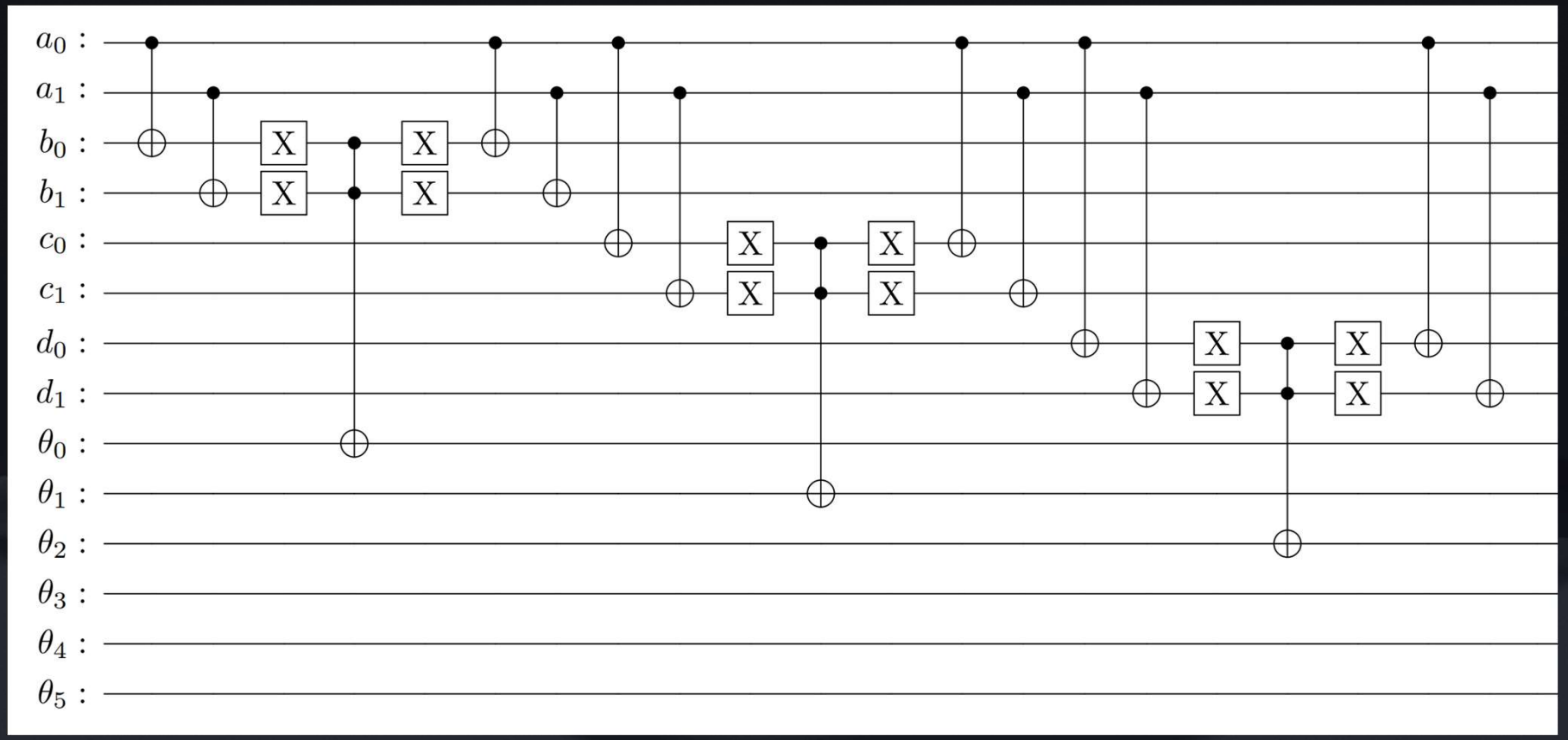
Comparator circuit

$$\text{Comparator}(a, b, o) = \begin{cases} o \oplus 1, & \text{if } a = b; \\ o, & \text{if } a \neq b. \end{cases}$$



Positional exclusivity block

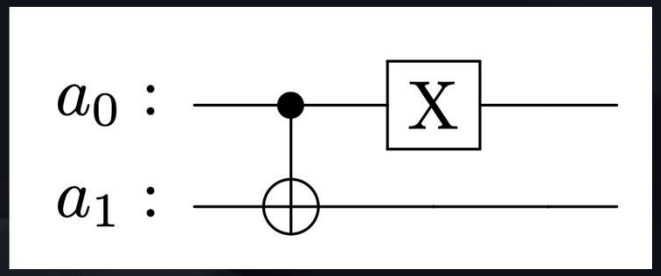
Each vertex should have a different position



Plus One and Minus One circuits

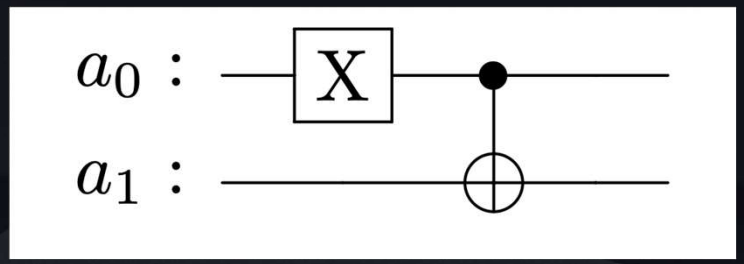
Plus one circuit

Takes state $|i> \; to \; state |(i+1) mod 2^n>$

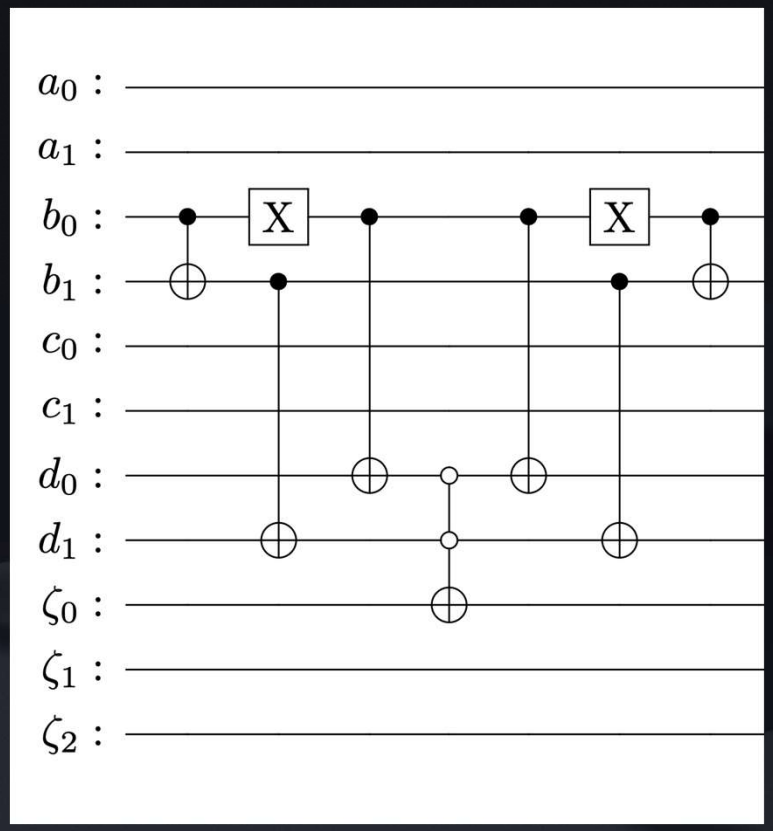


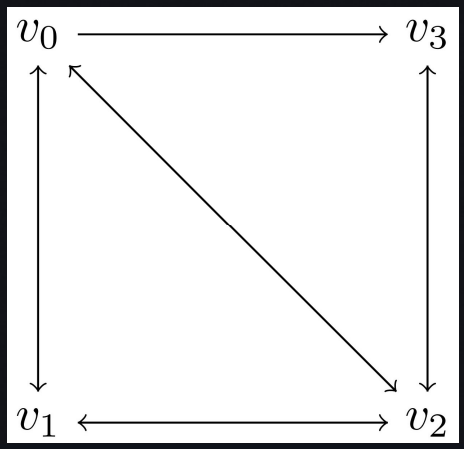
Minus one circuit

Takes state $|i> \; to \; state |(i-1) mod 2^n>$



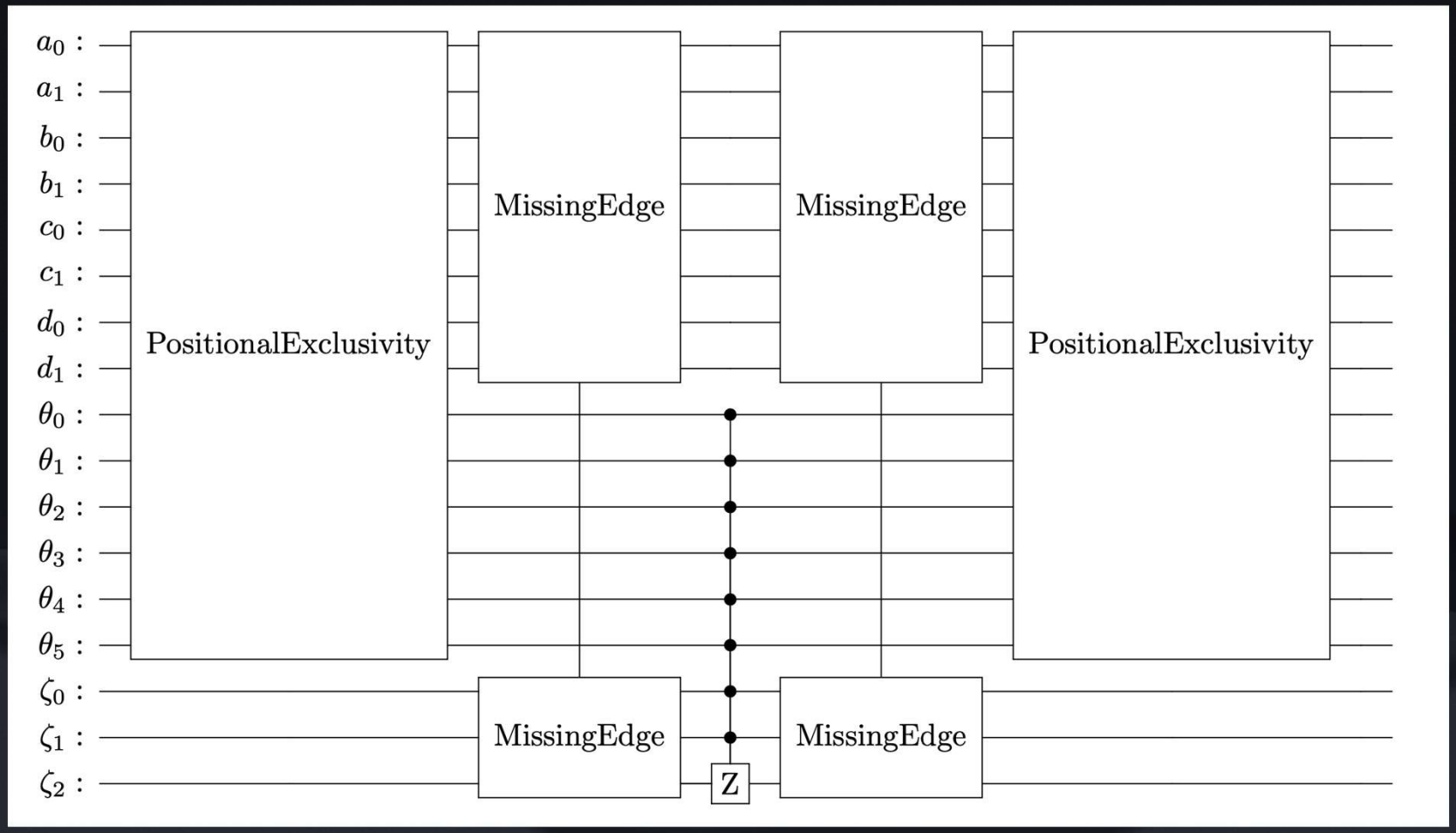
Missing Edge Block





Check if v3 comes after v1

Complete Oracle Design



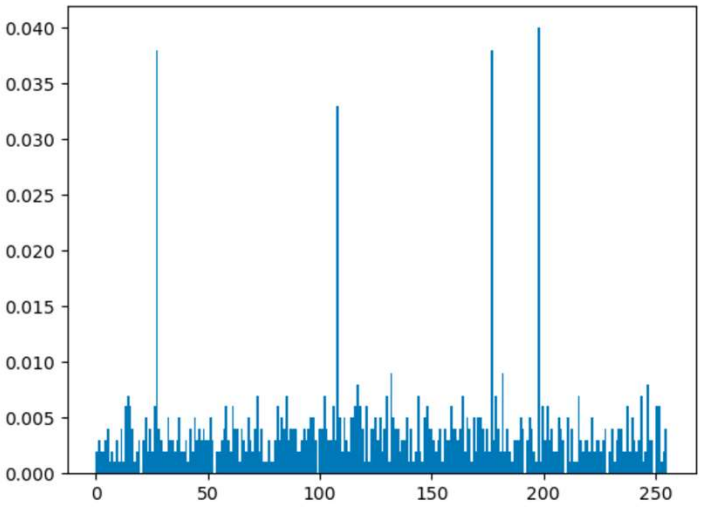


Fig. 11. Results of running the algorithm first sample graph with 1 iteration

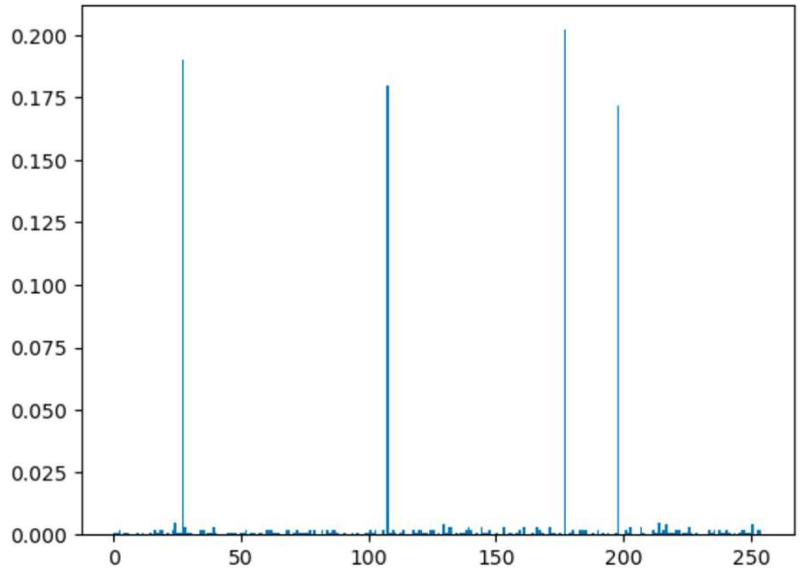
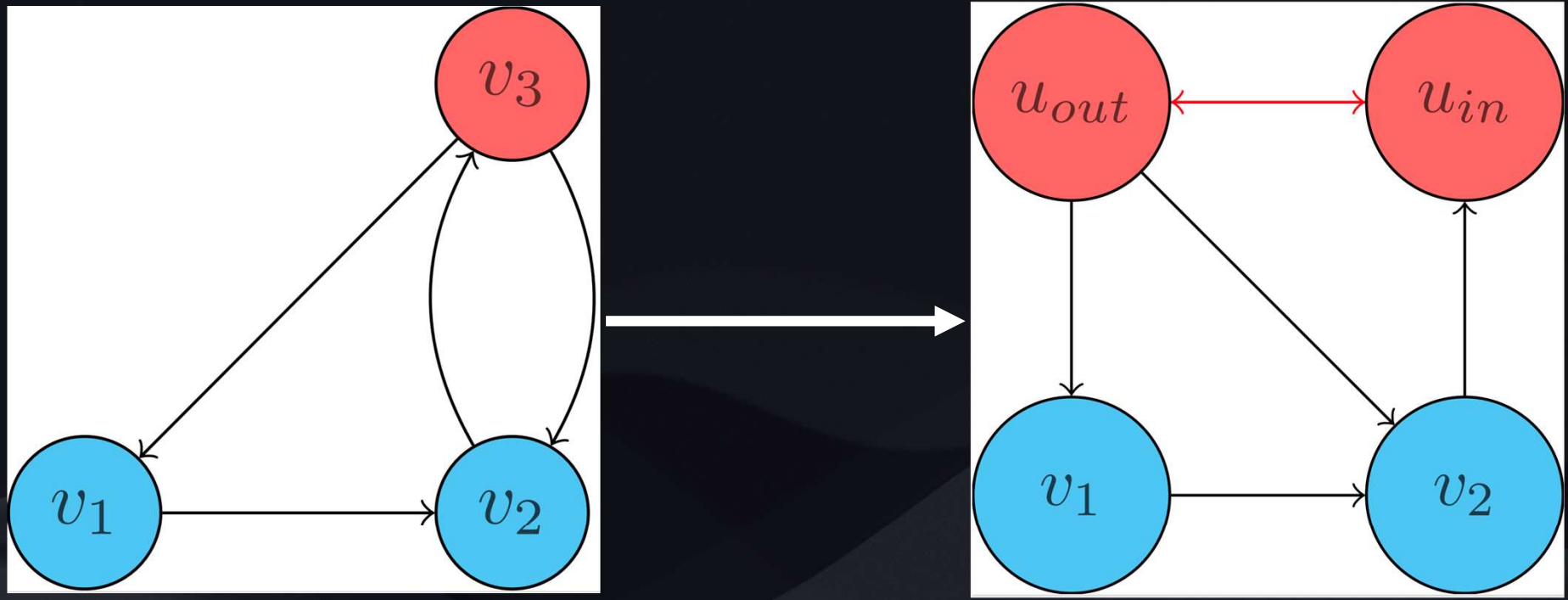
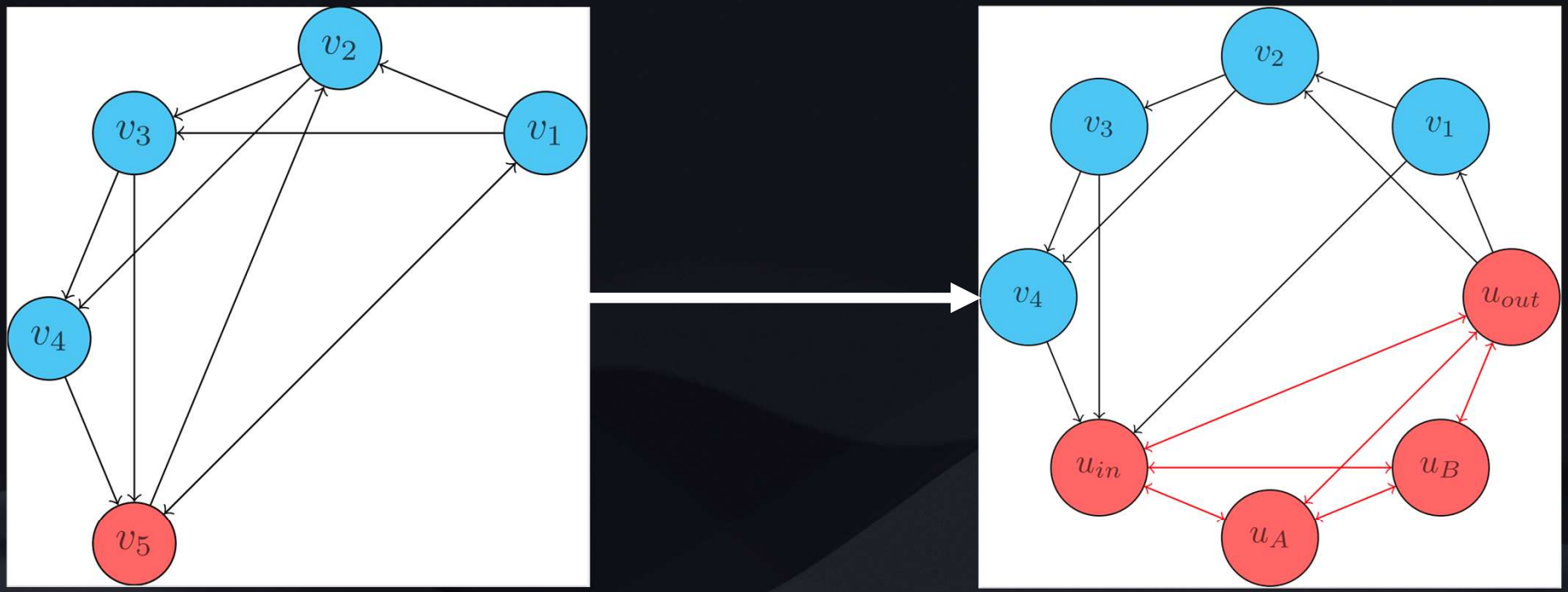


Fig. 12. Results of running the algorithm first sample graph with 8 iterations

Generalization for an arbitrary graph

- Inefficient Plus One Circuit in modulus p
- Adding Invalid position block


v_3
v_1
v_2
u_{out}
u_{in}
v_1
v_2


v_2
v_3
v_1
v_4
v_5
v_2
v_3
v_1
v_4
u_{out}
u_{in}
u_B
u_A

Conclusion

- Cuadratic advantage against brute force methods.
- Still in disadvantage against efficient heuristics.
- Needs technologies to scale.
- Possible subroutine for other algorithms.

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